

Azimuthal asymmetries in exclusive four-particle “Drell-Yan” events.

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Abstract. I study hard collisions between unpolarized protons and antiprotons where a lepton-antilepton pair is detected in coincidence with a final proton-antiproton pair, and no more particles are produced, in the regime $10 \text{ GeV}^2 \ll s \ll 1000 \text{ GeV}^2$, $M > 4 \text{ GeV}$, $q_T < 3 \text{ GeV}/c$. The present work is centered on azimuthal asymmetries. Because of momentum conservation, a Boer-Mulders term in the momentum distribution of a quark implies a balancing effect in the momentum distribution of some spectators. This produces azimuthal asymmetries of the final hadrons. To analyze this, I have organized a parton-level MonteCarlo generator where a standard $\cos(2\phi)$ -asymmetry of the dilepton distribution is produced, thanks to a soft rescattering process between an active quark coming from a hadron and a spectator anti-diquark coming from the other hadron. This produces $\cos(2\phi)$ -asymmetries of the final hadron pair. Hadron and lepton asymmetries have the same size.

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1 Introduction

The rising interest for the so-called spin-physics in inclusive hadronic reactions (see e.g. [1] or [2]) has among its consequences the planning of Drell-Yan experiments[3,4] at intermediate energies (squared c.m. energy $s \ll 1000 \text{ GeV}^2$). One is the PANDA experiment[5] where protons and antiprotons collide with variable $s \leq 30 \text{ GeV}^2$ (see [6] for an extensive description of the experiment goals and methods, and [7] for a shorter summary). This experiment has a broad spectrum of objectives. These include unpolarized Drell-Yan, finalized at a precision measurement of the dilepton $\cos(2\phi)$ -asymmetry for $s \approx 30 \text{ GeV}^2$, and dilepton mass $M > 2 \text{ GeV}/c^2$.

The $\cos(2\phi)$ -asymmetry, or ν -asymmetry, was first discussed in [8,9]. It was found nonzero, with size 5-20 %, in π^- -nucleus experiments [10,11,12,13,14], where s was between 100 and 600 GeV^2 and both x and $\bar{x} \sim 0.1$. It was recently found small or zero in the pp experiment[15] at much larger s and much smaller average x , \bar{x} . PANDA will measure it in a peculiar large- x regime, with large statistics. Both muon pairs and electron pairs will be detected.

Apart for specific kinematics, another peculiarity distinguishes PANDA from all the previous fixed-target Drell-Yan experiments. Because of the multi-purpose nature and design of this apparatus, it will detect the class and momentum of almost all of the reaction products.

In the past, the most appealing feature of the Drell-Yan measurement (in the case of dimuon detection) has just been the need not to care other particles apart for the $\mu^- \mu^+$ pair. With a thick layer of hadron-absorbing material and muon spectrometers downstream, the structure of a fixed-target Drell-Yan apparatus is relatively simple (see [16,17] for reviews). For this reason, we have very little information about what is produced in Drell-Yan measurements, in coincidence with the dilepton pair. The author is only aware of charge multiplicity measurements at ISR[18,19], at the large $\sqrt{s} \sim 100 \text{ GeV}$ (where a large number of particles was produced, of course).

To analyze what PANDA could find concerning Drell-Yan fragments, in a previous work[20] I presented a simulation of Drell-Yan events at $s = 30 \text{ GeV}^2$, with minimum dilepton mass $2 \text{ GeV}/c^2$ and minimum transverse momentum $0.8 \text{ GeV}/c$. That simulation was performed with Pythia-8[21]. I found some surprising results:

- 1) Almost all the events contain one (and only one) nucleon-antinucleon pair.
- 2) 50 % of the events *only* contains a nucleon-antinucleon pair (apart for the dilepton, of course). These pairs are equally divided into $p\bar{p}$ and $n\bar{n}$ pairs.
- 3) In most of the other events the $N\bar{N}$ pair is accompanied by not more than two light particles (charged pions or photons, possibly from π^0 decay).
- 4) An insertion of the simulated events into the PANDA acceptance shows that in the case of “dilepton + dipro-

ton” events with a detected dilepton, half of the $p\bar{p}$ pairs will be fully detected. This is about 10 % of the total rate of detected Drell-Yan events in PANDA conditions.

5) In another 10 % of detected Drell-Yan events we will only detect a single proton (often) or a single antiproton (more rarely), but we will be able to identify a “dilepton + diproton” exclusive event.

This opens the possibility to analyze 20 % of the Drell-Yan events as *exclusive* events, with some interesting associated observables and with the perspective of accessible modeling (because of the small number of involved particles). This work will be centered on an analysis of the “dilepton + diproton” events, including azimuthal asymmetry effects. Clearly, the standard properties of these events may be simulated via Pythia or some other known multi-purpose code as I have already done in [20]. Here my primary goal is to analyze those unknown properties of the proton-antiproton pair, associated with the dilepton azimuthal $\cos(2\phi)$ -asymmetry. To this aim, I have organized a specific parton-level generator code.

The extra detected proton-antiproton pair enriches the analysis with two more vectors: \mathbf{p}_P and $\mathbf{p}_{\bar{P}}$. These may be combined into the two pairs $\mathbf{p}_P \pm \mathbf{p}_{\bar{P}}$. In 4-particle events only the relative momentum $\mathbf{p}_P - \mathbf{p}_{\bar{P}}$ is independent of the dilepton momentum \mathbf{q} (that is equal to the sum of the lepton momenta and to minus the sum of the fragment momenta). So, here I will focus on the observables related with the correlations among the fragment relative momentum, the lepton relative momentum and \mathbf{q} .

Although my main interest is towards PANDA, I will consider a hypothetical experiment with $s = 100 \text{ GeV}^2$ instead of 30 GeV^2 as in PANDA (a possibility in this kinematic range could be Drell-Yan at COMPASS[22]). This will allow me to use several simplifying approximations, based on the fact that the longitudinal momenta of the hadron constituents are for most events much larger than the transverse momenta. In the PANDA case this is not true, and the analysis would require nonstandard devices, and infrared parameters.

1.1 Notations

I will use the letter “ p ” to indicate observable momenta (leptons, proton, antiproton) and the letter “ k ” for the momenta of the partons (quark, diquark, antiquark, antidi-quark). In detail, p_μ and \bar{p}_μ indicate lepton momenta, k_μ and \bar{k}_μ (anti)quark momenta, while for the other particles a suffix is present.

The vector q_μ is the sum of the lepton momenta, and also of the quark momenta, and minus the sum of the spectator momenta and of the final hadron momenta. I use both $q = \sqrt{q_\mu q^\mu}$ and M to indicate the same thing, i.e. the invariant mass of the dilepton pair.

The generic letters θ and ϕ indicate all the polar and azimuthal angles used in this work. The text and the figure captions specify each time which angles I am speaking about (suffixes resulted in difficult to read figure captions).

Results will be presented with respect to angles measured in two reference frames:

a) “hadron collision frame”: the center of mass frame of the colliding proton and antiproton, with z along the proton direction. The other two axes are fixed once and for all, so this frame is the same for all events. The fragment momenta are always referred to this frame. The lepton momenta are referred to this frame when I look for correlations between them and the hadron momenta.

b) “co-oriented dilepton c.m. frame”: this is an event-dependent frame with all axes parallel to the former one, but boosted in such a way to be a center of mass frame for the lepton-antilepton pair. This frame is used to calculate the azimuthal $\cos(2\phi)$ -asymmetry of the leptons, in a way that is as much similar to the tradition as possible. In the present case, the main reason to avoid the Collins-Soper[23] frame and of other similar frames with event-dependent orientation of the axes is the difficulty in relating lepton and hadron properties.

Another frame appears in the intermediate stages of the calculation:

c) “spin frame”: an event-dependent frame, whose origin and z -axis coincide with those of the hadron collision frame, and whose x and y axes are random-rotated with flat distribution around the z -axis. The quark and anti-quark spins are projected along the y -axis of this frame, but no initial or final observable quantity refers to it.

2 General ideas

I have built a “Drell-Yan-based” MonteCarlo model: as much as possible of the knowledge of the inclusive Drell-Yan process at partonic level has been used here. This includes the leading twist factorization properties[24,25,26,27], the x -distributions and k_T -distributions as they have been measured in this process and in the required regime[16,17], the phenomenology and theory of the inclusive lepton $\cos(2\phi)$ asymmetry[8,9,10,11,12,13,14,15,28,29,30]. This should not hide the fact that I want to reproduce something that is not exactly a Drell-Yan process. In the individual channels composing the inclusive Drell-Yan process, higher twist effects may be much stronger than in the overall sum, and factorization properties have never been discussed.

In the hadron-quark-spectator vertex I assume a basic diquark form for the spectator, according with the widespread quark-diquark model used in [31]. I will only consider one quark flavor (according to [20] 90 % of the proton-antiproton Drell-Yan events derive from $u\bar{u}$ annihilations, so this is a good approximation).

For the hadronization process, I will consider two alternative possibilities: (i) I will assume that spectator momenta may be identified with final hadron momenta, (ii) I will assume that the final $q\bar{q}$ pair, needed to allow the spectators to form physical hadrons, is created with a reasonably distributed relative momentum.

My main goal is to reproduce properties of the final proton-antiproton pair, that have the same physical origin of the dilepton azimuthal $\cos(2\phi)$ -asymmetry.

In the pioneering works by [8,9] the $\cos(2\phi)$ -asymmetry is one of the terms appearing in the most general form

for the dilepton pair distribution in unpolarized Drell-Yan. Within the scheme given in [28], this asymmetry is rewritten according with the quark-parton model, where it is proportional to the product of two terms (one for the quark, one for the antiquark) of the form $h(x, k_T) \mathbf{s}_T \wedge \mathbf{k}_T$, where $h(x, k_T)$ is the Boer-Mulders function. A commonly agreed definition for it may be found in [32], and some models exist [30, 33, 34, 35, 36].

The $q\bar{q} \rightarrow \gamma^*$ coupling selects $q\bar{q}$ pairs with opposite helicities. Combined with the Boer-Mulders spin-momentum correlation, this selects $q\bar{q}$ pairs with nontrivial space distribution properties. The virtual photon is able to transfer some of these properties to the dilepton relative momentum, resulting in the known $\cos(2\phi)$ -asymmetry.

To understand what this could mean for the spectator distribution, I trace this path back to its origin: the $\cos(2\phi)$ -asymmetry derives from a property of the splitting vertex according to which a quark has nonzero $\langle \mathbf{s}_T \wedge \mathbf{k}_T \rangle$. The spectators must balance this with $\langle \mathbf{s}_T \wedge \mathbf{p}_{T, \text{spect}} \rangle = -\langle \mathbf{s}_T \wedge \mathbf{k}_T \rangle$, since in the splitting vertex $\mathbf{p}_{T, \text{spect}} + \mathbf{k}_T = \mathbf{P}_{T, \text{parent}} = 0$. This should imply azimuthal asymmetries of the final proton and antiproton, for the subset of events where these are observed in coincidence with a dilepton pair.

The spin-momentum correlation picture of [28] poses some fundamental problems that have been solved at theoretical level [30], but there is some ambiguity about how to implement this theory in a parton MonteCarlo model. To clarify this, some general premise is useful:

(i) In a theoretical model for a quantum process the expectation values of some variables are not obliged to have a physical meaning in the intermediate steps. In a MonteCarlo they should better have, since all the steps are classical. (ii) Loop integrals often present relevant final cancellations. In a MonteCarlo a single event is only one of the infinite configurations entering a loop. Sometimes cancellations take place anyway over a large set of events, sometimes they do not. (iii) In a theoretical calculation much is gauge artifact. In a MonteCarlo, only physical particles and interactions can appear. (iv) Interference processes do not admit a straightforward implementation. (v) A “black box” distribution that turns around problems always exists, but one would like to avoid it, as much as possible.

The product of two spin-momentum correlations, each one implying $\langle s_y k_x \rangle \neq 0$ for the quark and the antiquark respectively, does not violate general invariance laws. However, each of the two is individually unphysical. In [30] the ultimate reason for the Boer-Mulders spin-momentum correlation is searched in initial state interactions between active and spectator partons originating from different parent hadrons. A nontrivial reduction work shows that it is possible to rewrite these effects as a distortion of the quark and antiquark initial-state momentum distributions, in a factorized format.

This suggests two possible schemes for a MonteCarlo, that I name “strict factorization scheme” and “rescattering scheme”. In the former one implements the conclusion of [30], in the latter the starting point of the same work. In

other words, in one case one assumes that the stationary momentum distribution of a quark in a hadron presents unphysical spin-momentum correlations, in the other case that this is the effect of interactions with partons coming from another hadron.

At leading twist and given some constraints on the rescattering interaction, the two schemes lead to the same results (for the observable variables discussed in this work), for the following two reasons:

(i) Since the works on leading-twist factorization in Drell-Yan [24, 25, 26, 27] it is known that gluon exchanges in initial state interactions are in large part gauge artifacts, or cancel in loops, or may be reabsorbed in the distribution functions, or belong to a class of phenomena that have no effect in the limit of high collision energy. Translated into an implementation of rescattering, this means that only exchanges of transverse momentum at a fixed scale are admitted (a few GeV/c at most). This also implies $k_T \ll k$ for all the partons.

(ii) At leading twist, the $q\bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$ cross section (with assigned transverse polarizations for q and \bar{q}) is symmetric for quark-antiquark exchange if the quark momenta respect $k_T \ll k$ (see eq. 9 and the discussion in the Appendix). As a consequence, an event where an amount of transverse momentum is exchanged between a spectator and its companion quark, and an event where this momentum is exchanged between the same spectator and the oppositely coming antiquark, lead to the same observable final state variables.

I will work in the rescattering scheme. It allows for organizing a chain of probabilistic steps, none of which is a priori unphysical.

Another problem is posed by the fact that rescattering is the effect of an interference process. The probability of the considered process may be written in the form

$$Prob \sim |(S * P * C + S * A * C)|^2. \quad (1)$$

Here S is the amplitude for the splitting process of the two hadrons and may be further factorized as $S(1)S(2)$, P describes the free propagation of all the partons, A is first order rescattering, C is the hard $q\bar{q} \rightarrow l^+ l^-$ process, and “*” indicates sum/convolution with respect to the degrees of freedom of the partons in the intermediate stages.

My assumption is that for any given spin configuration of the $q\bar{q}$ pair in the intermediate state I may rewrite

$$\begin{aligned} |(S * P * C + S * A * C)|^2 &\rightarrow |S|^2 * |P + A|^2 * |C|^2 \\ &= |S(1)|^2 |S(2)|^2 * |P + A|^2 * |C|^2(2) \end{aligned}$$

and substitute $|P + A|^2$ with a physically allowed scattering process between physical particles.

An interaction that is suitable for this aim is the “scalar + spin-orbit” one, well known in hadron scattering (see e.g. [37]). Its effect in the scattering of a spin-1/2 projectile by an unpolarized target is to produce a final momentum distribution of the form $a(k_T) + b(k_T) \mathbf{s}_T \wedge \mathbf{k}_T$. It does not violate any invariance law, derives from an interference process between scattering and no-scattering amplitudes and modifies the quark momentum distribution

in the same way as an “intrinsic” Boer-Mulders function would do.

What is done here is the probabilistic version of the quantum method used in [40], where one rewrites at parton level the hadron-hadron spin-orbit interactions. The method has some overlap with the 1-gluon exchange used in perturbative models[30,33,34,35,36].

Alternatively, spin-orbit rescattering may be just considered an effective way to distort the quark momentum distribution, so to introduce a Boer-Mulders term $\propto s_T \wedge k_T$ with pre-assigned shape, without worrying about its physical explanation in more fundamental terms.

3 Constraints from the features of inclusive Drell-Yan

A detailed discussion of the following equations may be found in a very extensive form in e.g.[38], and in more concise and specific form in [14,29], or in many other works.

Neglecting quark spin effects and assuming one relevant quark flavor, the parton model $P\bar{P} \rightarrow l^+l^- + X$ cross section is

$$\sigma = A \frac{u(x, k_T)u(\bar{x}, \bar{k}_T)}{x\bar{x}s} \frac{H_{\mu\nu}L^{\mu\nu}}{q^4}. \quad (3)$$

Here A is a normalization constant, the factor $1/x\bar{x}s \approx 1/q^2$ combines flux and phase space factors, $u(x, k_T)$ is the u -quark unpolarized distribution in an unpolarized proton. $L^{\mu\nu}$ is the lepton tensor, $H_{\mu\nu}$ is the $q\bar{q}$ tensor, defined so that $H_{\mu\nu}L^{\mu\nu}/q^4$ is adimensional.

The quark and antiquark variables entering this equation are x, \bar{x}, k_T and \bar{k}_T . From the measurement of the lepton momenta \mathbf{p} and $\bar{\mathbf{p}}$ we may reconstruct x, \bar{x} and $\mathbf{q}_T = \mathbf{p}_T + \bar{\mathbf{p}}_T = \mathbf{k}_T + \bar{\mathbf{k}}_T$. The individual quark transverse momenta are not observable, but some features of the distribution of $\mathbf{k}_T - \bar{\mathbf{k}}_T$ may be inferred from the distribution of $\mathbf{p}_T - \bar{\mathbf{p}}_T$.

To introduce quark spin effects in the generator I rewrite the above cross section in the form

$$\sigma \approx \sum_{s_y} \sum_{\bar{s}_y} A f(s_y) f(\bar{s}_y) \frac{u(x, k_T)u(\bar{x}, \bar{k}_T)}{x\bar{x}s} \frac{H_{\mu\nu}(s_y, \bar{s}_y)L^{\mu\nu}}{q^4} \quad (4)$$

where a random axis y is chosen, and the polarizations s_y, \bar{s}_y are random sorted with distribution $f(s_y)$.

Eq.4 gives the same results as eq.3, as far as the momentum and the spin distributions $u(x, k_T)$ and $f(s_y)$ are uncorrelated. The spin-orbit rescattering, or equivalently the Boer-Mulders terms, introduces a correlation. In this case the phenomenological distribution is

$$d\sigma \propto \frac{1 + \cos^2(\theta)}{x\bar{x}s} \left(u(x, k_T)u(\bar{x}, \bar{k}_T) + h(x, k_T)h(\bar{x}, \bar{k}_T)\sin^2(\theta)\cos(2\phi) \right) d[\cos(\theta)]d\phi. \quad (5)$$

(see sections 4.4 and 5.1 for the definition of θ and ϕ).

The simulation code is parameterized in such a way that the final outcome reproduces the features of eq.5 and of the related phenomenology for the dilepton distributions in π^-W fixed target experiments[10,11,12,13,14]. We have not data on the $\cos(2\phi)$ asymmetry in $p\bar{p}$ Drell-Yan, so I have to rely on π^-W data. In both cases we have dominance of valence-valence $u\bar{u}$ annihilations.

4 MonteCarlo structure

The event are sorted at $s = 100 \text{ GeV}^2$. Quarks and leptons are considered light-like. With regard to spectators, we only need their 3-momentum, so it is not relevant to specify their mass. All the events are subject to the cutoff $\tau > 4^2/100$, where $\tau \equiv x\bar{x} \approx M^2/s$.

4.1 General flowchart

The chain of steps in the simulation is:

1) The proton with initial momentum $P_0\hat{z} \approx \sqrt{s}/2\hat{z}$ splits into a light-like quark and a diquark. The quark has 3-momentum $[k_x, k_y, xP_0]$ that is sorted randomly: x is sorted according to a distribution $f(x)/x$; k_x and k_y refer to the random-oriented spin frame, and are sorted with gaussian distribution with center at $k_x = k_y = 0$. Details on the parameters are given later. The factor $1/x$ in $f(x)/x$ is needed to take into account a factor $1/x\bar{x}$ in the cross section (eq.3).

2) The 3-momentum of the spectator diquark is set to $[-k_x, -k_y, (1-x)P_0]$.

3) A value $s_y = \pm 1$ (meaning y -spin = $\pm\hbar/2$) is sorted for the quark, with probability 50% vs 50%. At this stage, the sorting of the spin and of the other variables are independent. The diquark is assumed a scalar one, so it has no spin. As far as final state rescattering is neglected, the difference between scalar and vector diquarks is not relevant.

4) The same operations (1, 2, 3) are performed for the antiproton, whose initial momentum is $-P_0\hat{z}$.

5) Spin-orbit shift of the quark momentum: $k_x \rightarrow k_x + s_y\Delta k_x$. The shift Δk_x is gaussian-distributed, with positive average value of $\langle \Delta k_x \rangle$. After this step the quark has $\langle k_x \rangle = s_y \langle \Delta k_x \rangle$.

6) The anti-diquark (i.e. the spectator coming from the antiproton) is subject to the opposite shift: its initial transverse momentum $\bar{k}_{d,x} = -\bar{k}_x$ is shifted to $-\bar{k}_x - s_y\Delta k_x$. The result of this and of the previous step is that the total momentum of the quark and of the anti-diquark is conserved.

7) Steps (5,6) are symmetrically performed on the \bar{q} -diquark pair.

8) The quark and the antiquark variables including spins are used to build the hadron tensor.

9) In the lepton co-oriented c.m. frame (see section 1.1), for q^2 fixed by the quark and antiquark momenta, the angles specifying the direction of the lepton momenta are sorted isotropically. After transforming them to the spin frame, the lepton tensor is build.

10) The lepton and the hadron tensor are contracted, and the scalar $W(s_y, \bar{s}_y) \equiv H^{\mu\nu}(s_y, \bar{s}_y)L_{\mu\nu}/q^4$ is used in an accept/reject procedure, where the event “ $q\bar{q}$ transformed into l^-l^+ ” is accepted or rejected with probability $\propto W(s_y, \bar{s}_y)$ (W does not depend on q^2 , but only on the relative orientation of the momenta). If the event is rejected, one restarts from step (1) and all the variables have to be sorted again.

11) In the case of an accepted event, the observable momenta (still in the spin frame) are transformed to the hadron collision frame or to the lepton co-oriented c.m. frame for the data analysis.

4.2 k_T -distributions and $s_x k_y$ distortion

Here I discuss the sorting of the quark variables. For the antiquark, the procedure is the same.

The initial components of \mathbf{k}_T are sorted with gaussian distribution, with $\langle k_x \rangle = \langle k_y \rangle = 0$, and $\sqrt{\langle k_T^2 \rangle} = 0.4$ GeV/c.

The shift Δk_x is sorted randomly with quasi-gaussian distribution with $\langle \Delta k_x \rangle = 0.5$ GeV/c and fluctuation 0.27 GeV/c. “Quasi-gaussian” means that we have the boundary conditions -1 GeV/c $< \Delta k_x < 2$ GeV/c, and that the sorting probability tends regularly to zero at these limits. The value 0.27 GeV/c has the only motivation of reproducing the phenomenological data of [10,11,12,13,14].

The spin-orbit effect for quarks from a proton and antiquarks from an antiproton is here assumed to have the same sign of $\langle (\mathbf{s}_T \wedge \mathbf{k}_T) \cdot \mathbf{P}_{parent} \rangle$. This means that $\langle (\mathbf{s}_T \wedge \mathbf{k}_T) \cdot \hat{z} \rangle$ is opposite in the two cases, since the two parent hadrons are opposite-directed.

4.3 x -distribution

The x -distribution has the basic form $f(x) = x^a(1-x)^b$. Fixed-target Drell-Yan experiments at $s \sim 100$ -600 GeV² used the same parameterization and gave $a \approx 0.5$, $b \approx 2$ (see e.g. the appendix of [14]). I have adopted these two values.

It must be noted that the lower limit $s x \bar{x} > (4 \text{ GeV})^2$ weakens the experimental constraints on a , and decreases the relevance of a precise value for it.

4.4 Polarized quark – lepton tensor contraction

The calculation of this tensor-tensor contraction is reported in the Appendix. For steps (8,9,10) of the previous flowchart I need the probability distribution

$$W(s_y, \bar{s}_y) \equiv H^{\mu\nu}(s_y, \bar{s}_y)L_{\mu\nu}/q^4 \quad (6)$$

where $L_{\mu\nu}$ is the lepton tensor and $H^{\mu\nu}(s_y, \bar{s}_y)$ is the tensor associated to a quark and an antiquark with assigned transverse polarizations.

Extracting a factor q from each vector:

$$k^\mu \equiv (q/2)N^\mu, \quad p^\mu \equiv (q/2)n^\mu, \dots \quad (7)$$

I get

$$W_{unpol} = \frac{1}{8} [(Nn)(\bar{N}\bar{n}) + (N\bar{n})(\bar{N}n)]. \quad (8)$$

and

$$W_{s_y \bar{s}_y} = [1 + (s_y \bar{s}_y)] \cdot W_{unpol} - \frac{1}{2} s_y \bar{s}_y (1 + n_y \bar{n}_y) \quad (9)$$

with $s_y = \pm 1$. To get to this result, terms with magnitude k_T/q have been systematically neglected. So eq.9 is a leading twist result.

The first term in eq.9 is about 3 times more relevant than the second one in determining the overall cross section. In the center of mass of the $q\bar{q} \rightarrow l^+l^-$ annihilation, where the virtual photon is at rest, and the 3-vectors \mathbf{n} , \mathbf{N} , etc, are unitary, the unpolarized term W_{unpol} favors configurations where the quark and the lepton directions are aligned, according to the known “ $1 + \cos^2(\theta)$ ” law.

In fig.1 I show the distribution $f(|\cos(\theta)|)$ together with a fitting curve $\propto [1 + \cos^2(\theta)]$ where θ is the lepton polar angle in the lepton co-oriented c.m. frame. In the Collins-Soper frame we should get a precise $[1 + \cos^2(\theta)]$ -law, and for large s the z -axes of two frames coincide. Fig.1 shows that we are very close to this condition.

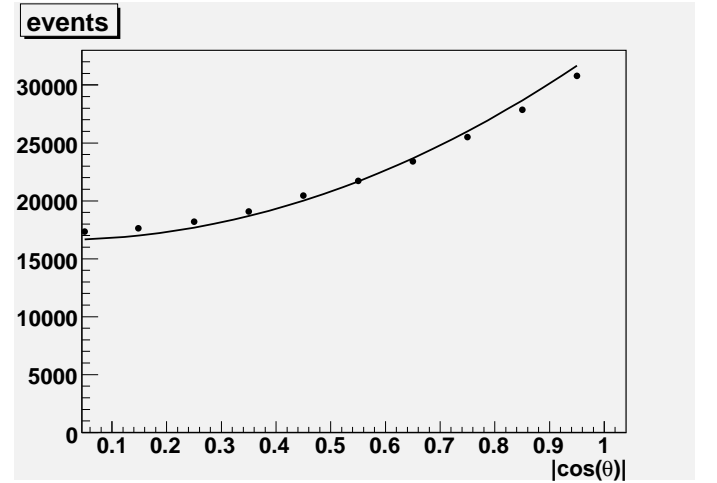


Fig. 1. Event distribution w.r.t. the polar angle of the positive lepton in the lepton co-oriented c.m. frame. The continuous curve is a fit of the form $N = N_0[1 + \cos^2(\theta)]$. For each point, a very small error bar is present although not visible, estimated by the standard fluctuation.

5 Results

I evaluate the measurable asymmetries according to the definition

$$A_{\pm} \equiv \frac{N(+) - N(-)}{N(+) + N(-)} \quad (10)$$

where $N(\pm)$ is the simulated number of events corresponding to a positive or negative value of some observable.

In most of the following cases the observable is a factor $\cos(2\phi)$, where ϕ is a lepton or hadron azimuthal angle ranging from 0 to 2π . I also present figures with event distributions vs ϕ , and hand-made fits of the form

$$f(N, \beta, \phi) \equiv N[1 + \beta \cos(2\phi)] \quad (11)$$

For a distribution exactly coinciding with eq.11, the asymmetry calculated according with eq.10 is

$$A_{\pm} = \frac{2}{\pi}\beta \approx \frac{2}{3}\beta \quad (12)$$

A typical value that I find is $\beta \approx 0.2$ corresponding to $A_{\pm} \approx 0.13$.

All the following simulations have been performed once more after removing any spin-momentum correlation. Because of statistical fluctuation, or of uncontrollable numeric systematic effects, these “vacuum” simulations show asymmetries of magnitude 2 %. So we know that few-percent magnitudes cannot be taken seriously given the event numbers used in the following.

5.1 Behavior of the lepton $\cos(2\phi)$ –asymmetry

In this subsection ϕ is the angle between the projections on the transverse xy plane of the vector $\mathbf{q} = \mathbf{p} + \bar{\mathbf{p}}$ measured in the hadron c.m. frame, and the vector $\mathbf{p} - \bar{\mathbf{p}}$ measured in the lepton co-oriented c.m. frame (where it simply coincides with $2\mathbf{p}$). This is equivalent to the ordinary definition of the angle appearing in the $\cos(2\phi)$ –asymmetry, although the traditional procedure is to rotate the lepton c.m. frame in such a way that ϕ is just the angle between the lepton transverse momentum and the x –axis. Some slightly different choices are normally adopted for the z –axis, whose implications have been discussed in [39]. In [14] the data have been organized according to different frames, and one may appreciate the differences.

In fig.2 I show the ϕ –distribution for the events with q_T between 1.5 GeV/c and 3 GeV/c. The fitting curve is $1 + 0.23 * \cos(2\phi)$ (apart for an overall normalization).

The applied rescattering model does not introduce directly any x –dependence or q_T –dependence: all the quarks are (statistically) subject to the same spin-dependent momentum shift. However, indirectly a strong q_T –dependence and some x –dependence is produced in the final results.

In Table 1 I report the event numbers and the asymmetry of a set of simulations corresponding to a series of consecutive bins in q_T . The event numbers in this table are proportional to the corresponding cross sections. For these event sets a further cutoff has been applied on the polar angle: only events with $|\theta| < 45^\circ$ have been counted in this statistics. Apart for enhancing the $\cos(2\phi)$ –asymmetry, the second choice is useful to avoid the problems described in [41], and similar cutoffs have been adopted largely in the previously quoted experiments because of small-angle and large-angle acceptance problems.

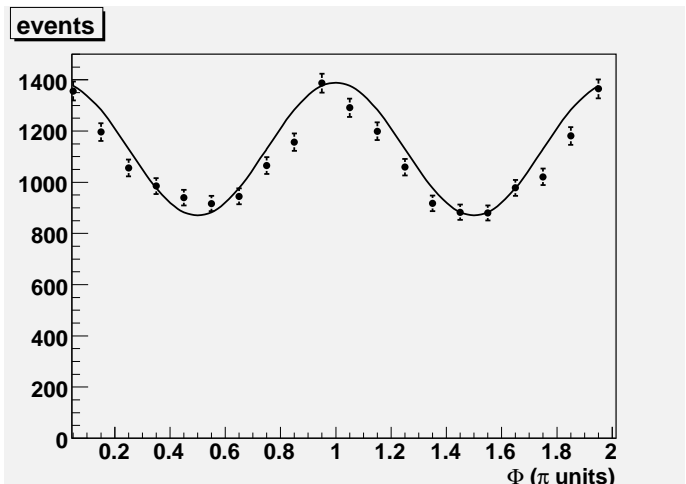


Fig. 2. ϕ –distribution of the simulated dilepton events, with the transverse momentum of the virtual photon satisfying $1.5 \text{ GeV/c} < q_T < 3 \text{ GeV/c}$. For the definition of ϕ see the text. The fit is $\propto 1 + 0.23 * \cos(2\phi)$.

Range (GeV/c)	Event difference	Event sum	Asymmetry
0-0.5	163	32564	0.005
0.5-1	1093	56314	0.02
1-1.5	2035	33826	0.06
1.5-2	2061	12677	0.16
2-2.5	872	3435	0.25
2.5-3	208	616	0.34

Table 1: Distribution w.r.t. q_T of dilepton events and of their $\cos(2\phi)$ –asymmetry, with the general cutoff $|\tan(\theta)| < 1$. For the definition of the ϕ –angle, see the text.

The chosen parameters of the distorting term produce a $\cos(2\phi)$ –asymmetry with a reasonable magnitude and q_T –dependence (see the data in [10,11,12,14], and the fit in [29]), at least up to 2.5 GeV/c.

The experiments [10,11,12,14] show a decreasing trend at large x , and give no information for x below the valence region. The proton-proton measurement by E866[15] had larger beam energy and smaller average x and \bar{x} , and involved one sea active quark at least. It showed near-zero asymmetries.

My simulations show a decrease of the asymmetries for large and small x , \bar{x} . It is however difficult to disentangle the role of the two. To properly analyze the small- x case I should remove the limit $q > 4 \text{ GeV/c}^2$ on the dilepton mass. This would take me into the region $q_T/q \approx 1$, where I cannot rely on some approximations used in building the simulation code. Also, near the edges of the phase space the physics may be seriously affected by constraints that I have not applied here, and by higher twist terms. So I would not take seriously simulations in the regions where x and/or \bar{x} are close to zero or one. In the other regions, the asymmetries are roughly x –independent.

5.2 Proton-antiproton $\cos(2\phi)$ –asymmetry

In Table 2, I report final hadron asymmetries and compare them with the corresponding lepton asymmetries in two relevant kinematic situations. For the hadron $\cos(2\phi)$ asymmetry, ϕ is the angle between the difference and the sum of the transverse momenta of the proton and of the antiproton, in the hadron collision c.m. frame (remark: the sum is $-\mathbf{q}_T$). For the lepton $\cos(2\phi)$ –asymmetry, the angle is the same as in the previous section. Apart for the transverse momentum cuts, and for the cutoff $4 \text{ GeV}/c^2$ on the dilepton mass, no further cuts have been applied to these simulations.

Pair	Range (GeV/c)	Event diff.	Event sum	Asymmetry
leptons	1.5 - 3	17921	133498	0.1342
hadrons		17847	133498	0.1336
leptons	0 - 1.5	16763	974911	0.017
hadrons		48550	974911	0.050

Table 2: Compared $\cos(2\phi)$ –asymmetries for leptons and hadrons

For the first case of Table 2, the ϕ –distribution of the hadron pairs is reported in fig.3. The fitting curve is $1 + 0.22 * \cos(2\phi)$ (apart for an overall normalization).

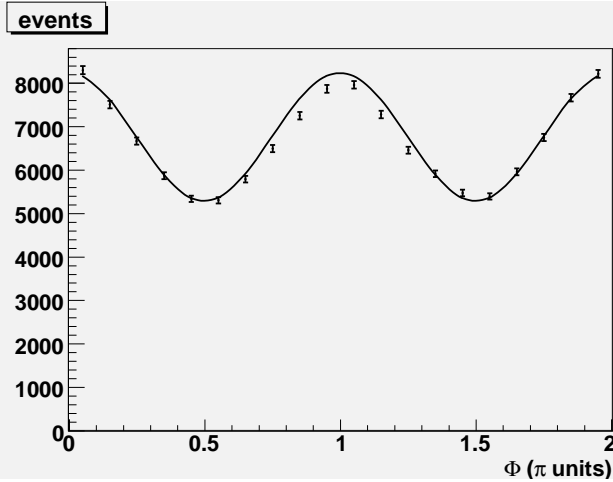


Fig. 3. Distribution of the proton-antiproton pairs w.r.t. the angle ϕ between the sum and difference of the hadron transverse momenta in the hadron collision frame. These events are the same as in Table 2, upper part, with cutoff $1.5\text{--}3 \text{ GeV}/c$ for the total transverse momentum and $4 \text{ GeV}/c^2$ for the dilepton mass. The fit is $\propto 1 + 0.22 * \cos(2\phi)$.

5.3 hadron-lepton correlations

At leading twist the hadron and the lepton vectors cannot communicate through quantities that are odd in the charge exchange on one of the two sides (hadron or lepton). I have checked the correlations $(p_x \bar{p}_y - p_y \bar{p}_x)_{lept} (p_x \bar{p}_y -$

$p_y \bar{p}_x)_{hadr}$, and $(\mathbf{p} - \bar{\mathbf{p}})_{lept} \cdot (\mathbf{p} - \bar{\mathbf{p}})_{hadr}$ (transverse components), for which I have found nothing meaningful.

The ϕ –angle between the relative transverse momentum of the leptons and of the hadrons could present ϕ –even systematic behaviors, since it is the difference between ϕ_{lept} and ϕ_{spect} when both are measured in the same frame. However, the ϕ –distribution in fig.4 does not show reliable systematic deviation from homogeneity. I remark that in fig.4 the vertical event scale starts from 5000. If it started from zero as in the other figures, it would appear almost completely flat. The fitting curve is $1 + 0.03 * \cos(2\phi)$ (apart for an overall normalization). This corresponds to a possible asymmetry about 2 %. This magnitude is within the simulation uncertainties and is not meaningful.

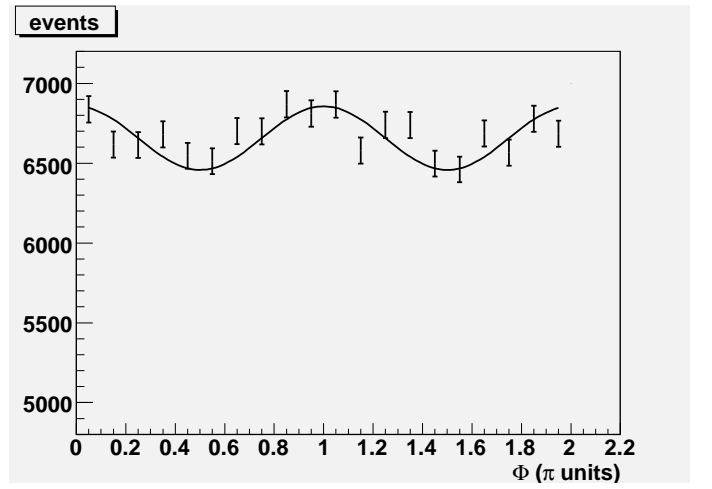


Fig. 4. Event distribution w.r.t. the angle ϕ between the relative transverse momentum of the diproton pair and the relative transverse momentum of the dilepton pair, both referred to the hadron collision frame. Same cuts as in fig.3. The fit is $\propto 1 + 0.03 * \cos(2\phi)$.

5.4 Single hadron - leptons correlations

A strong correlation is the one between one of the hadron fragments (e.g. the proton) and the lepton momentum $(\mathbf{p} - \bar{\mathbf{p}})_{lept}$ (transverse components).

From a theoretical point of view this adds nothing new, because the momentum of any of the spectator partons may be written as $-\mathbf{q}/2 \pm (\mathbf{p} - \bar{\mathbf{p}})_{spect}/2$, so any observable that is linear with respect to the momentum of a spectator is not independent from those ones that I have considered previously. From the experimenter’s point of view, however, to measure a proton-dilepton asymmetry in the laboratory frame is easier than detecting both a proton and an antiproton (the leptons must be detected in any case). According with the analysis in [20], in the specific case of PANDA we have a large number of events where it is possible to identify a $\mu^+ \mu^- p \bar{p}$ –event from the detection of μ^+ , μ^- and p only.

In fig.5 I report the distribution of the ϕ -angle between the proton momentum and the difference between the lepton momenta (all in the hadronic center of mass frame). The only cutoff is $q_T > 1.5$ GeV/c. The corresponding $\cos(2\phi)$ -asymmetry is 0.14 to be compared to the value of the lepton asymmetry that is 0.13. The fit is $\propto 1 + 0.22 * \cos(2\phi)$.

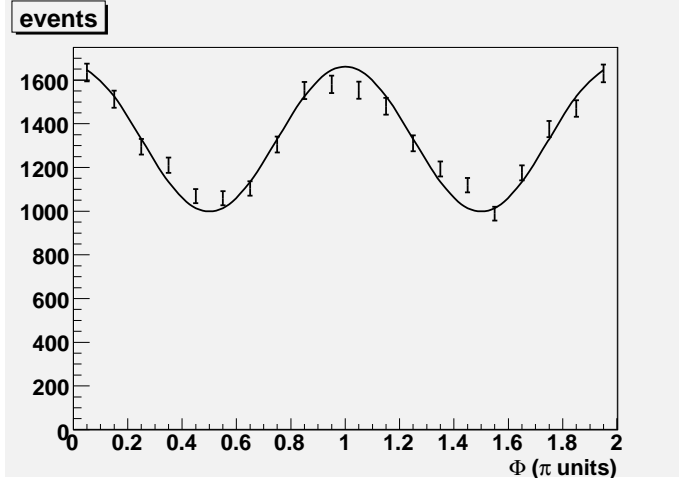


Fig. 5. Event distribution w.r.t. the angle ϕ between the proton transverse momentum and the dilepton relative momentum in the hadron collision frame. Same cuts as in figs.3 and 4. As remarked in the text, the evident asymmetry is just another way to measure the ordinary dilepton $\cos(2\phi)$ -asymmetry. The fit is $\propto 1 + 0.22 * \cos(2\phi)$.

6 Higher twist and hadronization effects

A very simple setup for a realistic hadronization is the one used by Pythia in the small phase-space limit of the string fragmentation model: the creation of a single $q\bar{q}$ -pair with a random relative transverse momentum (see the detailed description in section 2.4.1 of [42], and eq.25 in [43]). I have simulated the effect of this on the distribution of the final hadron pair.

Gaussian width of the relative P_T due to hadronization	Asymmetry
0	0.134
0.35 GeV/c	0.121
0.7 GeV/c	0.091

Table 3: Changes of the hadron $\cos(2\phi)$ -asymmetry associated with a relative gaussian-distributed transverse momentum, due to the $q\bar{q}$ pair that forms the final hadrons together with the diquarks coming from the initial state. 0.134 is the reference value from Table 2.

In other words, my previous simulation relative to table 2 (upper line) has been repeated with the addition of a

gaussian broadening of the relative transverse momentum of the final hadron pair. I have tested two cases, where the gaussian broadening has width 0.35 and 0.7 GeV/c. The results for the hadron $\cos(2\phi)$ -asymmetry are reported in table 3.

Since 0.2-0.4 GeV/c is a reasonable range for the width, we see that the hadronization process, at least in this simple form, does not change the hadron azimuthal asymmetry.

Some authors (see [44,45,46]) have proposed modifications of this basic form of the string model, so to include spin-dependent mechanisms in a 2-hadron production stage. Probably, this class of processes would more seriously affect the hadron asymmetries calculated by me, but they cannot touch the lepton asymmetries. So, the approximate equality of lepton and hadron asymmetries, predicted in this work, is a signature of the absence of final state spin-dependent effects.

The process $p\bar{p} \rightarrow \mu^+\mu^- + p\bar{p}$ has chances to be affected by higher-twist effects, since it is an exclusive process. In a scheme where at least approximate factorization is present and higher twist terms are a correction and not the main part of the cross section, these effects may be separated into initial-state interactions (that link active and spectator partons) and final-state interactions (not implying direct or indirect momentum exchange between spectator and active partons).

Higher-twist effects in the initial state should preserve the approximate equality of hadron and lepton asymmetries (although the shape of both could differ from what I suggest here). Indeed, the momentum-conservation mechanism that is behind the dilepton-dihadron correlation is independent of the details of the initial-state processes.

Summarizing: lepton $\cos(2\phi)$ -asymmetries that are in disagreement with my predictions (fig.2, table 1) are a signature of higher twist effects in general, while the inequality of hadron and lepton asymmetries is a signature of specific spin-dependent final state effects.

7 Conclusions

This work has considered exclusive $p\bar{p} \rightarrow \mu^+\mu^- + p\bar{p}$ events at $s = 100$ GeV². The main focus is (i) the possibility of a lepton $\cos(2\phi)$ -asymmetry, of the same nature of the one observed in inclusive $\mu^+\mu^- + X$ events at the same energy, (ii) the possibility of associated asymmetries of the recoiling hadrons.

The model simulations presented here show that $\cos(2\phi)$ asymmetries of the leptons in their c.m. frame should be accompanied by $\cos(2\phi)$ asymmetries of the hadrons in the laboratory frame. In the latter case, ϕ is the angle between the difference and the sum of the hadron transverse momenta. The size of these asymmetries is the same.

If the measured asymmetries differ from what I have suggested here, this is a signature of higher twist effects (since my predictions for the lepton distributions may be read as a re-parameterization of the known distributions measured in inclusive Drell-Yan). But, if the lepton and hadron asymmetries differ strongly between them, this is

a more specific signature of final state spin-dependent effects.

A Appendix: Polarized quark–lepton

contraction $H^{\mu\nu}(s_y, \bar{s}_y)L_{\mu\nu}$.

We need the contraction

$$q^4 W \equiv H^{\mu\nu} L_{\mu\nu} \quad (13)$$

of the quark-level hadronic and lepton tensors in the process $q\bar{q} \rightarrow l^+l^-$, where all four particles are ultrarelativistic. I use the shortened notation for the traces

$$T[.] \equiv \frac{1}{4} \text{Tr}[..]. \quad (14)$$

I use the definitions of the Berestevskij-Lifits-Pitaevskij book[47] (better known as the 4th book of the Landau-Lifits Course in Theoretical Physics). As the only exception to this, I indicate $k_\mu \gamma^\mu$ with the widespread notation \not{k} instead of using \hat{k} as was done in that book.

A.1 The case of unpolarized quarks

If the quarks are unpolarized, we simply get

$$\begin{aligned} q^4 W_{unpol} &\equiv T[\not{k}\gamma^\mu \bar{k}\gamma^\nu] T[\not{p}\gamma_\mu \bar{p}\gamma_\nu] = \\ &= 2 [(kp)(\bar{k}\bar{p}) + (k\bar{p})(\bar{k}p)]. \end{aligned} \quad (15)$$

I extract a factor q from each vector:

$$k^\mu \equiv (q/2)N^\mu, \quad p^\mu \equiv (q/2)n^\mu, \dots \quad (16)$$

getting

$$W_{unpol} = \frac{1}{8} [(Nn)(\bar{N}\bar{n}) + (N\bar{n})(\bar{N}n)]. \quad (17)$$

In a center of mass frame of the partonic process, the 3-vectors \mathbf{N} , \mathbf{n} etc are unitary vectors. In the hadron collision frame this property is not true, but of course the full tensor contraction is frame-independent so W_{unpol} is a scale-independent object. The same is valid for the leading-twist terms in the spin-dependent $W(s_y, \bar{s}_y)$ that will be calculated below.

A.2 polarized quarks (leading twist)

In the case of a polarized (anti)quark

$$\not{k} \rightarrow \not{k}(1 - \gamma^5 \not{s}) \quad (18)$$

where s^μ is the polarization 4-vector for the quark, respecting the exact 4-dimensional constraint $(sk) = 0$. If the spin is strictly transverse to the particle motion, then also $\mathbf{s} \cdot \mathbf{k} = 0$.

In the implementation of a multi-particle MonteCarlo, it would not be practical to define a spin quantization

axis that accompanies each individual quark in its movements, so I have assigned a common quantization axis (the y -axis) for all spins. Since here quarks present nonzero k_T components, this means $O(k_T/q)$ helicities.

The relation between s^μ and the polarization in a rest frame σ is, for massive particles,

$$s_0 = \sigma_L |\mathbf{k}|/m, \quad s_L = \sigma_L E/m, \quad \mathbf{s}_T = \sigma_T \quad (19)$$

and evidently it creates problems for U.R. particles, unless the longitudinal component is strictly zero. However, when the previous expressions are used to write the density matrix eq.18 in terms of the rest frame polarizations, E/m -terms cancel and we are left free from mass singularities:

$$\not{k}(1 - \gamma^5 \not{s}) = \not{k}[1 - \gamma^5(\pm\sigma_L + \sigma_T \cdot \gamma_T)]. \quad (20)$$

(\pm differentiates particles and antiparticles). This says that a way exists to make calculations without meeting mass singularities.

In the following it will be comfortable to use invariant trace formalism. So, it is useful to understand which apparently large terms may appear and how to treat them.

In the product of two $(1 - \gamma^5 \not{s})$ density matrices with not exactly transverse spins, special care is needed with the terms containing the product of one or two longitudinal components. The former give zero trace for odd parity of the number of gamma matrixes. The latter modify W_{unpol} to

$$W_{h,\bar{h}} = (1 - h\bar{h})W_{unpol}. \quad (21)$$

where h and \bar{h} are the average helicities due to the longitudinal projections of the quasi-transverse spin. The magnitude of these components is

$$h \sim k_y/q, \quad \bar{h} \sim \bar{k}_y/q, \quad (22)$$

that leads to $W_{h,\bar{h}} = W_{unpol} + \text{higher twist terms}$, systematically neglected here.

The full trace for polarized quarks is

$$H_{s,\bar{s}}^{\mu\nu} \equiv T[\not{k}\gamma^\mu(1 - \gamma^5 \not{s}) \bar{k}\gamma^\nu(1 - \gamma^5 \not{\bar{s}})] \quad (23)$$

$$= T[\not{k}\gamma^\mu \bar{k}\gamma^\nu] - T[\not{k}\gamma^\mu \not{s} \bar{k}\gamma^\nu \not{\bar{s}}]. \quad (24)$$

This gives a sum of 15 terms. Two of them contain the products $(sk) = 0$ and $(\bar{s}\bar{k}) = 0$. We are left with

$$\begin{aligned} H_{s,\bar{s}}^{\mu\nu} &= \left([1 - (s\bar{s})] \cdot H_{unpol}^{\mu\nu} - \frac{q^2}{2} \{s^\mu, \bar{s}^\nu\} \right) + \\ &+ \left((k\bar{s})\{k^\mu, \bar{s}^\nu\} + (\bar{k}s)\{\bar{k}^\mu, s^\nu\} - (k\bar{s})(\bar{k}s)g^{\mu\nu} \right) \end{aligned} \quad (25)$$

where $\{a_\mu, b_\nu\} \equiv a_\mu b_\nu + a_\nu b_\mu$.

After contracting the hadron tensor with the lepton tensor, the final result consists of many terms. I drop the $h\bar{h}$ -terms on the ground of eq. 22. The transverse spin dependent terms contain the product $q^4 s_y \bar{s}_y$, and have forms like e.g. $q^4 s_y \bar{s}_y N_y \bar{n}_y (\bar{N}n)$. I neglect systematically

all the products that contain one at least among N_x , N_y , \bar{N}_x , \bar{N}_y . These terms do not reach the magnitude q^4 , since $N_x \sim k_x/q$ etc.

I retain all the terms containing $(ns)(\bar{n}\bar{s})$, $(n\bar{s})(\bar{n}s)$, $(s\bar{s})$ (transverse components of n and \bar{n} with magnitude ~ 1 are statistically frequent).

The final result is

$$W_{s_y \bar{s}_y} = [1 + (s_y \bar{s}_y)] \cdot W_{unpol} - \frac{1}{2} s_y \bar{s}_y (1 + n_y \bar{n}_y). \quad (26)$$

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